HPC simulations of cardiac electrophysiology using patient-specific models of the human heart CompBioMed & VPH Institute webinar

Francesc Levrero-Florencio & Ana Mincholé

Computational Cardiovascular Science Group Department of Computer Science, University of Oxford









Contents

- Introduction to the heart (electro)physiology
- Mathematical modelling
- CHASTE
- Alya
- Final Comments

Brief introduction to (electro)physiology

Introduction to the physiology of the heart

- Heart is a contracting muscle
- 4 chambers
- 2 systems: pulmonary and systemic



{https://commons.wikimedia.org}

Electrophysiology of the heart

- SA node Atria AV node brief delay - His bundle branches down the septum
- Purkinje fibers allow propagation throughout the endocardium
- Cell to cell propagation through gap junctions.









Zacur et al, BIVPCS 2017

Cell electrophysiology

- Four phases of the action potential:
 - ✓ Upstroke (0/1)
 - ✓ Plateau (2)
 - ✓ Repolarisation (3)
 - ✓ Resting potential (4)
- Calcium and Sodium currents are involved during the upstroke and plateau phases
- Potassium-based currents are involved in the repolarisation



{Ravens U and Cerbai E. Europace 2008}

Tissue electrophysiology



{Essentials of Human Physiology 2010}

Wellcome Images

Cardiac modelling

Body

Body surface potentials ECG recordings mHealth data

Organ

Clinical EP studies Multi-modality clinical MRI

Tissue

High-resolution MRI Histology Optical mapping Cell cultures

Single cell

Microelectrode recordings Protein/mRNA expression Optical mapping

Ionic current

Voltage/patch clamp Isolated cells/hiPSC-CMs





 $\label{eq:propagation model} \begin{array}{|c|c|} \hline Propagation model \\ \hline \nabla \cdot \overline{\sigma}_i \nabla \Phi_i = \beta \cdot I_m - I_{st_i} \\ \hline \nabla \cdot \overline{\sigma}_e \nabla \Phi_e = -\beta \cdot I_m \end{array}$









$I_{Na} = g_{Namax} * m^{3*}h^*j^*(V_m - E_{Na})$ $\frac{dx}{dt} = \alpha_x(1 - x) - \beta_x x$ $\alpha_x = \alpha_x(V_m); \beta_x = \beta_x(V_m)$



Mathematical modelling

First cardiac AP model



Denis Noble (1936-present)



November 5, 1960 NATURE NO 4749 495Cardiac Action and Pacemaker Potentials based on the Hodgkin-Huxley Equations D. NOBLE Phil. Trans. R. Soc. Lond. B 307, 353-398 (1985) [353] Printed in Great Britain A MODEL OF CARDIAC ELECTRICAL ACTIVITY INCORPORATING IONIC PUMPS AND CONCENTRATION CHANGES By D. DIFRANCESCO¹ AND D. NOBLE, F.R.S.² ÷ Ca²⁺ 3Na Na, Ca b, Na b,Ca

... to current human AP models



Monodomain and bidomain models

Monodomain model $\nabla \cdot (\mathbf{D}_0 \nabla V) = \chi (C \frac{\mathrm{d}V}{\mathrm{d}t} + I_{ion} - I_{app})$

Bidomain model $\nabla \cdot (\mathbf{D}_{int} \nabla V) + \nabla \cdot (\mathbf{D}_{int} \nabla U) = \chi (C \frac{\mathrm{d}V}{\mathrm{d}t} + I_{ion} - I_{app})$ $\nabla \cdot [(\mathbf{D}_{int} + \mathbf{D}_{ext}) \nabla U] + \nabla \cdot (\mathbf{D}_{int} \nabla V) = 0$





{Adebisi et al. J Biomed Sci Eng. 2013}

The ratio of the intracellular and extracellular conductivity tensors;

| Bidomain: | Monodomain: | $\mathbf{D}_i = \begin{pmatrix} D_{\parallel}^i \\ 0 \\ 0 \end{pmatrix}$ | $\begin{array}{c} 0\\ D^t_{\perp}\\ 0 \end{array}$ | $\begin{pmatrix} 0 \\ 0 \\ D_{\perp}^{i} \end{pmatrix}$ |
|--|--|--|--|---|
| $\frac{D^{i}_{\parallel}}{D^{e}_{\parallel}} \neq \frac{D^{i}_{\perp}}{D^{e}_{\perp}}$ | $\frac{D^i_{\mathbb{I}}}{D^e_{\mathbb{I}}} = \frac{D^i_{\perp}}{D^e_{\perp}} = \alpha$ | $\mathbf{D}_{e} = \begin{pmatrix} D_{ }^{e} \\ 0 \\ 0 \end{pmatrix}$ | $\begin{array}{c} 0 \\ D^{e}_{\perp} \\ 0 \end{array}$ | $\begin{pmatrix} 0 \\ 0 \\ D_{\perp}^{e} \end{pmatrix}$ |

Cardiac tissue is more accurately described as a three-dimensional anisotropic *bidomain*, especially under conditions of applied external current such as in defibrillation studies.^[1-2]

B. J. Roth and J. P. Wikswo, IEEE Transactions on Biomedical Engineering 41, 232-240 (1994)
 J. P. Wikswo, et al., Biophysical Journal 69, 2195-2210 (1995)

Integrative physiology through modelling



Considered simulation software



- ✓ 0D,1D, 2D and 3D
- ✓ Petsc and Metis
- ✓ C++
- ✓ FE, BE, RK, CVODE
- ✓ Multi-scale simulation
- ✓ Highly scalable



DEPARTMENT OF COMPUTER SCIENCE



- ✓ 0D,1D, 2D and 3D
- ✓ In-house and Metis
- ✓ Modern Fortran
- ✓ FE
- ✓ Multiphysics
- ✓ Highly scalable (up to 100k cores)



Example of electrophysiology with Chaste

- Cardiac Chaste functionalities:
 - $\checkmark\,$ Monodomain and bidomain models



- Automatic implementation of cellular action potential models from the CellML repository
- ✓ Automatic generation of mathematical model for fibre orientation
- Checkpoint of simulations midway through run and restart with altered parameters
- ✓ Post-processing of simulation results to calculate electrophysiological properties such as action potential duration, conduction velocity, etc.

http://www.cs.ox.ac.uk/chaste/cardiac_index.html

2D electrical propagation using Chaste

• 2D Mesh geometry





Courtesy of Dr. Rina Ariga

Courtesy of Dr. Ernesto Zacur

Control 2D Electrical propagation



- 2D tissue from MRI
- O'Hara Rudy 2011 epicardial model
- Bidomain model
- Isotropic conductivities
- Regular stimulus of 600 ms

#include <cxxtest/TestSuite.h>
#include "PetscSetupAndFinalize.hpp"

#include "BidomainProblem.hpp"
#include "RegularStimulus.hpp"
#include "TetrahedralMesh.hpp"

#include "ORd2011epi_fkatpCvodeOpt.hpp"

class MyCellFactory : public AbstractCardiacCellFactory<2>
{
private:

boost::shared_ptr<RegularStimulus> mpStimulus;

public: MyCellFactory(float period) AbstractCardiacCellFactory<2>(), mpStimulus(new RegularStimulus(-120000.0, 1, period, 0)) r

AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode)

CellORd2011epi_fkatpFromCellMLCvodeOpt* p_cell;

double x = pNode->rGetLocation()[0]; double y = pNode->rGetLocation()[1];

if ((x>-0.32) δδ (x<-0.27) δδ (y>-0.1) δδ (y<0.1))

p_cell = new Cell0Rd2011epi_fkatpFromCellMLCvodeOpt(mpSolver, mpStimulus);

else

} }; p_cell = new Cell0Rd2011epi_fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);

return p_cell;

class TestConductivity : public CxxTest::TestSuite

void TestConductivity2d() throw(Exception)

// MESH
std::string filepath = "projects/anamin/test/data/2Dmesh";
DistributedTetrahedralWesh<2, >> mesh;
TrianglesMeshReader<2, >> mesh_reader(filepath);
mesh.ConstructForMeshReader(mesh_reader);

// NUMERICS
double odeT, pdeT, Print_Time;
odeT = 0.025;
pdeT = 0.05;
Print_Time=1; // ms

HeartConfig::Instance()->SetOdePdeAndPrintingTimeSteps(odeT, pdeT, Print_Time);

// BIOMAIN PARAMETERS
HeartConfig::Instance()->SetSurfaceAreaToVolumeRatio(1400);
HeartConfig::Instance()->SetCapacitance(1.0);
HeartConfig::Instance()->SetIntracellularConductivities(Create_cvector(1.2, 1.2));
HeartConfig::Instance()->SetExtracellularConductivities(Create_cvector(4, 4));

// SIMULATION TIME
HeartConfig::Instance()->SetSimulationDuration(1300); //ms

//POSTPROCESSING OPTIONS
// APD map
std::vector<std::pair<double,double> > apd map;
apd map.push_back(std::pair<double, double>(90.0, 0.0)); // APD90, 0 mV threshold
HeartConfig::Instance()-SetApdMaps(apd,map);

// Activation time map std::vector=double> upstroke time_map; upstroke time map.push.back(0.0); // 0 mV threshold HeartConfig::Instance()->SetUpstrokeTimeMaps(upstroke_time_map);

// Max Upstroke Velocity map
std::vector=double> maxupstroke_vel_map;
maxupstroke_vel_map.upsh_back(0.0);
HeartConfig::Instance()->SetMaxUpstrokeVelocityMaps(maxupstroke_vel_map);

// OUTPUT OPTIONS

// Oursol 0+1045 HeartConfig:Instance()->SetOutputDirectory(",/OUT"); HeartConfig:Instance()->SetOutputFilenamPrefix("results"); HeartConfig:Instance()->SetOutputFilenamPrefix("results"); HeartConfig:Instance()->SetVisualizeMitMParalNodeOrdering(true); HeartConfig:Instance()->SetVisualizeMitMParalLeVtK(true);

// PARAMETERS double period

MyCellFactory cell_factory(period);

// PROBLEM

}; }; BidomainProblem<2> bidomain_problem(&cell_factory);

= 600.0;

bidomain_problem.SetMesh(&mesh); bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();

| <pre>#include <cxxtest testsuite.h=""> #include "PetscSetupAndFinalize.hpp"</cxxtest></pre> | |
|--|----------------|
| #include "BidomainProblem.hpp" #include "RegularStimulus.hpp" #include "TetrahedralMesh.hpp" | |
| <pre>#include "ORd2011epi fkatpCvodeOpt.hpp"</pre> | |
| class MyCellFactory : public AbstractCardiacCellFactory<2> { private: boost::shared ptr≺RegularStimulus> moStimulus: | |
| <pre>public: MyCellFactory(float period)</pre> | |
| AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode) { | |
| <pre>CellORd2011epi_fkatpFromCellMLCvodeOpt* p_cell;</pre> | |
| <pre>double x = pNode->rGetLocation()[0]; double y = pNode->rGetLocation()[1];</pre> | |
| <pre>if ((x>-0.32) && (x<-0.27) && (y>-0.1) && (y<0.1)) { p_cell = new CellORd2011epi_fkatpFromCellMLCvodeOpt(mpSolver)</pre> | , mpStimulus); |
| <pre>clude "Picture of the provide and the provide provide and the provide provide and the provide provide and the provide pro</pre> | |
| | |
| <pre>class TestConductivity : public CoxTest::TestSuite { public: void TestConductivity2d() throw(Exception) { // MESH std::string filepath = "projects/anamin/test/data/2Dmesh"; DistributedTetrahedralWeshReader(filepath); mesh.constructFromWeshReader(filepath); mesh.constructFromWeshReader(filepath); // NMERICS double odeT, pdeT, Print_Time; odeT = 0.025; pdeT = 0.05; Print_Time]; // ms</pre> | |
| | |

HeartConfig::Instance()->SetOdePdeAndPrintingTimeSteps(odeT, pdeT, Print_Time);

// BLOWAIN PARAMETERS
HeartConfig::Instance()->SetSurfaceAreaToVolumeRatio(1400);
// L/cm
// uF/cm^2
HeartConfig::Instance()->SetIntracellularConductivities(Create_Cvector(1.2, 1.2));
HeartConfig::Instance()->SetEtracellularConductivities(Create_Cvector(1.4, 4));

// SIMULATION TIME
HeartConfig::Instance()->SetSimulationDuration(1300); //ms

//POSTRPCESSING OPTIONS
// APD map
std::vector<std::pair<double.double> > apd map;
std::vector<std::pair<double.double.doubles(90.0,0.0);
// APD90, 0 mV threshold
HeartConfig::Instance()->SetApdMap(ap(apd_map);

// Max Upstroke Velocity map
std::vectoredouble> maxupstroke_vel_map;
maxupstroke_vel_map.push_back(0.0);
HeartConfig::Instance().>SetMaxUpstrokeVelocityMaps(maxupstroke_vel_map);

// OUTPUT OPTIONS HeartConfig::Instance()->SetOutputDirectory(",/OUT"); HeartConfig::Instance()->SetOutputDirectory("results"); HeartConfig::Instance()->SetOutputDisngOriginalNodeOrdering(true); HeartConfig::Instance()->SetVisualizeWithParallelVtk(true); HeartConfig::Instance()->SetVisualizeWithParallelVtk(true);

HeartConfig::Instance()->SetVisualizeWithParallelVtk(t HeartConfig::Instance()->SetVisualizeWithParallelVtk(t // PARAMETERS double period = 600.0;

MyCellFactory cell_factory(period);

// PROBLEM
BidomainProblem<2> bidomain_problem(&cell_factory);

bidomain_problem.SetMesh(&mesh); bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();

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1:

class TestConductivity : public CxxTest::TestSuite

public: void TestConductivity2d() throw(Exception)

> // MESH std::string filepath = "projects/anamin/test/data/2Dmesh"; DistributedTetrahedralMesh<2,2> mesh; TrianglesMeshReader<2,2> mesh_reader(filepath); mesh.ConstructFromMeshReader(mesh reader);

// NUMERTCS double odeT, pdeT, Print Time; odeT = 0.025; pdeT = 0.05; Print_Time=1; // ms

HeartConfig::Instance()->SetOdePdeAndPrintingTimeSteps(odeT, pdeT, Print Time);

// BIDOMAIN PARAMETERS HeartConfig::Instance()->SetSurfaceAreaToVolumeRatio(1400); // 1/cm HeartConfig::Instance()->SetCapacitance(1.0); // uF/cm^2 HeartConfig::Instance()->SetIntracellularConductivities(Create c vector(1.2, 1.2)); HeartConfig::Instance()->SetExtracellularConductivities(Create c vector(4, 4));

// STMULATION TIME HeartConfig::Instance()->SetSimulationDuration(1300); //ms

//POSTPROCESSING OPTIONS // APD map std::vector<std::pair<double,double> > apd_map; apd_map.push_back(std::pair<double, double>(90.0, 0.0)); // APD90, 0 mV threshold HeartConfig::Instance()->SetApdMaps(apd_map);

// Activation time map std::vector<double> upstroke time map; upstroke time map.push back($\overline{0}$. $\overline{0}$); // 0 mV threshold HeartConfig::Instance()->SetUpstrokeTimeMaps(upstroke time map);

// Max Upstroke Velocity map std::vector<double> maxupstroke vel map; maxupstroke_vel_map.push_back(0.0); HeartConfig::Instance()->SetMaxUpstrokeVelocityMaps(maxupstroke_vel_map);

// OUTPUT OPTIONS HeartConfig::Instance()->SetOutputDirectory("./OUT"); HeartConfig::Instance()->SetOutputFilenamePrefix("results");

HeartConfig::Instance()->SetOutputUsingOriginalNodeOrdering(true); HeartConfig::Instance()->SetVisualizeWithMeshalyzer(false); HeartConfig::Instance()->SetVisualizeWithParallelVtk(true):

// PARAMETERS double period = 600.0:

MyCellFactory cell_factory(period);

// PROBLEM

BidomainProblem<2> bidomain_problem(&cell_factory);

bidomain problem.SetMesh(&mesh); bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();

#include <cxxtest/TestSuite.h> #include "PetscSetupAndFinalize.hpp"

#include "BidomainProblem.hpp" #include "RegularStimulus.hpp" #include "TetrahedralMesh.hpp"

#include "ORd2011epi_fkatpCvodeOpt.hpp"

```
#include <cxxtest/TestSuite.h>
#include "PetscSetupAndFinalize.hpp
#include "BidomainProblem.hpp
#include "RegularStimulus.hpp
#include "TetrahedralMesh.hpp
#include "ORd2011epi fkatpCvodeOpt.hpp
                                      class MyCellFactory : public AbstractCardiacCellFactory<2>
                public AbstractCardiacCel
class MyCellFactory :
.
orivate
  boost::shared_ptr<RegularStimulus> mpStimulu
                                      private:
ublic
  MyCellFactory(float period)
                                              boost::shared ptr<RegularStimulus> mpStimulus;
      : AbstractCardiacCellFactory<2>(),
       mpStimulus(new RegularStimulus(-120000
  AbstractCvodeCell* CreateCardiacCellForTissue public:
     CellORd2011epi_fkatpFromCellMLCvodeOpt*
                                              MyCellFactory(float period)
      double x = pNode->rGetLocation()[0];
                                                      : AbstractCardiacCellFactory<2>(),
      double y = pNode->rGetLocation()[1];
     if ( (x>-0.32) && (x<-0.27) && (y>-0.1)
                                                         mpStimulus(new RegularStimulus(-120000.0, 1, period, 0))
         p_cell = new CellORd2011epi_fkatpFrom
      else
        p_cell = new Cell0Rd2011epi_fkatpFrom
      return p_cell;
                                              AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode)
class TestConductivity : public CxxTest::TestSui
public:
  void TestConductivity2d() throw(Exception)
                                                     CellORd2011epi fkatpFromCellMLCvodeOpt* p cell;
     // MESH
     std::string filepath = "projects/ana
     DistributedTetrahedralMesh<2,2> mesh;
     TrianglesMeshReader<2,2> mesh_reader(file
                                                      double x = pNode->rGetLocation()[0];
     mesh.ConstructFromMeshReader(mesh reader
                                                      double y = pNode->rGetLocation()[1];
      // NUMERTCS
     double odeT, pdeT, Print Time;
     odeT = 0.025;
      pdeT = 0.05;
     Print_Time=1; // ms
                                                      if ((x > -0.32) \& (x < -0.27) \& (y > -0.1) \& (y < 0.1))
     HeartConfig::Instance()->SetOdePdeAndPrin
      // BIDOMAIN PARAMETERS
      HeartConfig::Instance()->SetSurfaceAreaTo
     HeartConfig::Instance()->SetCapacitance()
                                                             p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpStimulus);
     HeartConfig::Instance()->SetIntracellular
      HeartConfig::Instance()->SetExtracellula
      // STMULATION TIME
     HeartConfig::Instance()->SetSimulationDur
      //POSTPROCESSING OPTIONS
      // APD map
                                                      else
     std::vector<std::pair<double, double> > ap
apd map.push back(std::pair<double, doubl</pre>
      HeartConfig::Instance()->SetApdMaps(apd_m
      // Activation time map
                                                              p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);
     std::vector<double> upstroke time map;
      upstroke time map.push back(\overline{0}, 0):
      HeartConfig::Instance()->SetUpstrokeTime
     // Max Upstroke Velocity map
                                                      return p cell;
      std::vector<double> maxupstroke vel map
      maxupstroke_vel_map.push_back(0.0);
      HeartConfig::Instance()->SetMaxUpstrokeVe
     // OUTPUT OPTIONS
      HeartConfig::Instance()->SetOutputDirecto
      HeartConfig::Instance()->SetOutputFilenamePrefix(
      HeartConfig::Instance()->SetOutputUsingOriginalNodeOrdering(true);
      HeartConfig::Instance()->SetVisualizeWithMeshalyzer(false)
      HeartConfig::Instance()->SetVisualizeWithParallelVtk(true):
      // PARAMETERS
      double period
                       = 600.0:
     MyCellFactory cell_factory(period);
```

bidomain_problem.SetMesh(&mesh); bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();

BidomainProblem<2> bidomain_problem(&cell_factory);

// PROBLEM

| <pre>#include <cxxtest testsuite.h=""> #include "PetscSetupAndFinalize.hpp"</cxxtest></pre> | <pre>class TestConductivity : public CxxTest::TestSuite</pre> |
|---|---|
| <pre>#include "BidomainProblem.hpp" #include "RegularStimulus.hpp"</pre> | { |
| #include "TetrahedralMesh.hpp" | public: |
| <pre>#include "UK02011ep1_tkatpLv0deUp1.npp" class MvCellFactory : public AbstractCardiac()</pre> | <pre>void TestConductivitv2d() throw(Exception)</pre> |
| { private: | { |
| <pre>boost::shared_ptr<regularstimulus> mpStim public:</regularstimulus></pre> | // MESH |
| <pre>MyCellFactory(float period) : AbstractCardiacCellFactory<2>(),</pre> | <pre>std::string filenath = "projects/anamin/test/data/2Dmesh":</pre> |
| mpStimulus(new RegularStimulus(-120 { \ | DistributedTetrahedralMesh <2.25 mesh: |
| <pre>^ AbstractCvodeCell* CreateCardiacCellForTi</pre> | TrianglosMoshDoador<2.25 mosh roador(filopath): |
| <pre>{ CellORd2011epi_fkatpFromCellMLCvode0;</pre> | mosh ConstructEromMoshDoador/mosh roador); |
| <pre>double x = pNode->rGetLocation()[0]; double y = pNode->rGetLocation()[1];</pre> | mesh.constructrionmeshkeader (mesh_reader), |
| if ((x>-0.32) && (x<-0.27) && (y>-0. { p coll = per CollOPd2011epi fkatr | // NUMERICS |
| <pre>p_cert = new certowazorrepr_rwat; }</pre> | double odeT. pdeT. Print Time: |
| else { | odeT = 0.025: |
| <pre>} return p_cell;</pre> | pdeT = 0.05: |
| }; }; | Print Time=1: // ms |
| <pre>class TestConductivity : public CxxTest::Test { </pre> | |
| <pre>public: void TestConductivity2d() throw(Exception // </pre> | HeartConfig. Instance(). SetOdePdeAndPrintingTimeSteps(odeTpdeTPrint_Time). |
| <pre>// MESH std::string filepath = "projects/anan"</pre> | neur connig. instance() > secoder dexnar intingrimesteps(oder, pder, rrint_rime), |
| DistributedletrahedralMesh<2,2> mesh; TrianglesMeshReader<2,2> mesh_reader(mesh.ConstructFromMeshReader(mesh_rea | // RTDOMATH PARAMETERS |
| // NUMERICS | HoartConfig. Instance() SotSurfaceAreaTeVelumePatie(1400): // 1/cm |
| <pre>double ode1, pde1, Print_lime; odeT = 0.025; pdeT = 0.05;</pre> | HeartConfig. Instance() \sim SetCapacitance(1.0): |
| Print_Time=1; // ms | HeartConfig. Instance() ->SetCapacItance(1.0), // ur/cm 2 |
| HeartConfig::Instance()->SetUdePdeAnd | HeartConfig::Instance() ->SetIntracettularConductivities(Create_c_vector(0.1, 0.1)); |
| HeartConfig::Instance()->SetSurfaceAn HeartConfig::Instance()->SetCapacitan HeartConfig::Instance()->SetIntracel | Hear(config::Instance()->SetExtrace(tutarconductivities(create_c_vector(0.4, 0.4)); |
| HeartConfig::Instance()->SetExtracel | // SIMULATION TIME |
| HeartConfig::Instance()->SetSimulatic | HeartConfig::Instance()->SetSimulationDuration(1200): //ms |
| // APD map std::vector <std::pair<double.double></std::pair<double.double> | ······································ |
| apd_map.push_back(std::pair< double, d HeartConfig::Instance()->SetApdMaps(a | //POSTPROCESSING OPTIONS |
| <pre>// Activation time map std::vector<double> upstroke_time_mag</double></pre> | // APD map |
| upstroke_time_map.push_back(0.0); HeartConfig::Instance()->SetUpstrokeT | <pre>std::vector<std::pair<double.double> > and man:</std::pair<double.double></pre> |
| <pre>// Max Upstroke Velocity map std::vector<double> maxupstroke_vel_m</double></pre> | and man nuch back(std::nair <double, double="">(90.0.0.0)): // APD90 0 mV threshold</double,> |
| <pre>maxupstroke_vel_map.push_back(0.0); HeartConfig::Instance()->SetMaxUpstro</pre> | HeartConfig::Instance()->SetAndMans(and man): |
| | neur ceon rg, rrns cance() -> se chonops (aba_nab); |

// OUTPUT OPTIONS // WIND WIND HeartConfig: Instance()->SetOutputDirectory("./OUT"); HeartConfig: Instance()->SetOutputFilenamPrefix("results"); HeartConfig: Instance()->SetVotputFilenamPrefix("results"); HeartConfig: Instance()->SetVisualizeMitMPeshalyzer(false); HeartConfig: Instance()->SetVisualizeMitMPeshalUzer(false);

// PARAMETERS double period

MyCellFactory cell_factory(period);

// PROBLEM

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BidomainProblem<2> bidomain_problem(&cell_factory);

= 600.0;

bidomain_problem.SetMesh(&mesh); bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();

//POSTPROCESSING OPTIONS #include "PetscSetupAndFinalize.hpp // APD map #include "BidomainProblem.hpp" #include "RegularStimulus.hpp
#include "TetrahedralMesh.hpp std::vector<std::pair<double,double> > apd map; #include "ORd2011epi fkatpCvodeOpt.hpp' apd map.push back(std::pair<double, double>(90.0, 0.0)); // APD90. 0 mV threshold class MyCellFactory : public AbstractCardiacCellFac HeartConfig::Instance()->SetApdMaps(apd map); private: boost::shared_ptr<RegularStimulus> mpStimulus; public: MyCellFactory(float period) // Activation time map : AbstractCardiacCellFactory<2>(), mpStimulus(new RegularStimulus(-120000.0, std::vector<double> upstroke time map; upstroke time map.push back(0.0); // 0 mV threshold AbstractCvodeCell* CreateCardiacCellForTissueNo CellORd2011epi_fkatpFromCellMLCvodeOpt* p_ HeartConfig::Instance()->SetUpstrokeTimeMaps(upstroke time map); double x = pNode->rGetLocation()[0]; double y = pNode->rGetLocation()[1]; if ((x>-0.32) && (x<-0.27) && (y>-0.1) && // Max Upstroke Velocity map p_cell = new CellORd2011epi_fkatpFromCe std::vector<double> maxupstroke vel map; else maxupstroke vel map.push back(0.0); p_cell = new CellORd2011epi_fkatpFromCe HeartConfig::Instance()->SetMaxUpstrokeVelocityMaps(maxupstroke vel map); return p_cell; class TestConductivity : public CxxTest::TestSuite // OUTPUT OPTIONS ublic: void TestConductivity2d() throw(Exception) HeartConfig::Instance()->SetOutputDirectory("./OUT"); // MESH HeartConfig::Instance()->SetOutputFilenamePrefix("results"); std::string filepath = "projects/anamin/t
DistributedTetrahedralMesh<2,2> mesh; TrianglesMeshReader<2,2> mesh_reader(filepa HeartConfig::Instance()->SetOutputUsingOriginalNodeOrdering(true); mesh.ConstructFromMeshReader(mesh_reader); HeartConfig::Instance()->SetVisualizeWithMeshalvzer(false); // NUMERTCS double odeT, pdeT, Print Time; odeT = 0.025; pdeT = 0.05; HeartConfig::Instance()->SetVisualizeWithParallelVtk(true); Print_Time=1; // ms HeartConfig::Instance()->SetOdePdeAndPrinti // PARAMETERS // BIDOMAIN PARAMETERS HeartConfig::Instance()->SetSurfaceAreaToVo HeartConfig::Instance()->SetCapacitance(1.0 double period = 600.0: HeartConfig::Instance()->SetIntracellularCo HeartConfig::Instance()->SetExtracellularCo MyCellFactory cell factory(period); // STMULATION TIME HeartConfig::Instance()->SetSimulationDurat //POSTPROCESSING OPTIONS // PROBLEM // APD map std::vector<std::pair<double,double> > apd apd map.push back(std::pair<double, double> BidomainProblem<2> bidomain problem(&cell factory); HeartConfig::Instance()->SetApdMaps(apd_map // Activation time map std::vector<double> upstroke time map; upstroke time map.push back($\overline{0}, 0$): HeartConfig::Instance()->SetUpstrokeTimeMap bidomain problem.SetMesh(&mesh); // Max Upstroke Velocity map bidomain problem.SetWriteInfo(); std::vector<double> maxupstroke vel map: maxupstroke_vel_map.push_back(0.0); bidomain problem.Initialise(); HeartConfig::Instance()->SetMaxUpstrokeVelo bidomain problem.Solve(); // OUTPUT OPTIONS HeartConfig::Instance()->SetOutputDirectory HeartConfig::Instance()->SetOutputFilename HeartConfig::Instance()->SetOutputUsingOrig HeartConfig::Instance()->SetVisualizeWithMe HeartConfig::Instance()->SetVisualizeWithParallelVtk(true); // PARAMETERS double period = 600.0: MyCellFactory cell_factory(period); // PROBLEM BidomainProblem<2> bidomain_problem(&cell_factory); bidomain problem.SetMesh(&mesh);

#include <cxxtest/TestSuite.h>

1:

bidomain_problem.SetWriteInfo(); bidomain_problem.Initialise(); bidomain_problem.Solve();



- ✓ Similar APD values in all the geometry
- ✓ Activation time map starting from the septum
- ✓ Similar maximum upstroke velocities



Chaste example 2

• 2D propagation with ischemia



Ischemia in the anterior heart region:

- Hyperkalaemia: [K]o = 8.5 mM
 - Hypoxia: fkatp = 0.04
- Acidosis:
- \downarrow 25% g_{Na} and g_{CaL}



{Dutta et al, PBMB 2017}

```
class MyCellFactory : public AbstractCardiacCellFactory<2>
private:
   boost::shared ptr<RegularStimulus> mpStimulus;
    double k:
    double fkatp;
    double sf na ca;
public:
   MyCellFactory(float Input Ko, float Input fkatp, float SF Na Ca, float period)
        : AbstractCardiacCellFactorv<2>().
          mpStimulus(new RegularStimulus(-120000.0, 1, period, 0))
    ł
       k = Input Ko:
       fkatp = Input fkatp;
       sf na ca = SF Na Ca;
    AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode)
    {
       CellORd2011epi fkatpFromCellMLCvodeOpt* p cell;
       double x = pNode->rGetLocation()[0];
       double y = pNode->rGetLocation()[1];
       if ((x>-0.32) \& (x<-0.27) \& (y>-0.1) \& (y<0.1))
        {
            p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpStimulus);
        }
       else if ((x>-0.4) && (x<0.05) && (y>0.25) && (y<0.44))
            p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);
            p cell->SetParameter("extracellular potassium concentration",k);
           double gna = p cell->GetParameter("membrane fast sodium current conductance");
           double gca = p cell->GetParameter("membrane L type calcium current conductance");
           p cell->SetParameter("membrane fast sodium current conductance",gna*sf na ca);
           p cell->SetParameter("membrane L type calcium current conductance",gca*sf na ca);
           p cell->SetParameter("membrane atp dependent potassium current conductance",fkatp);
        }
       else
       {
            p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);
       }
```

{

```
//POSTPROCESSING OPTIONS
// APD map
std::vector<std::pair<double, double> > apd map;
apd map.push back(std::pair<double, double>(90.0, 0.0));
                                                             // APD90, 0 mV threshold
HeartConfig::Instance()->SetApdMaps(apd map);
// Activation time map
std::vector<double> upstroke time map;
upstroke time map.push back(0.0);
                                                             // 0 mV threshold
HeartConfig::Instance()->SetUpstrokeTimeMaps(upstroke time map);
// Max Upstroke Velocity map
std::vector<double> maxupstroke vel map;
maxupstroke vel map.push back(0.0);
HeartConfig::Instance()->SetMaxUpstrokeVelocityMaps(maxupstroke vel map);
// OUTPUT OPTIONS
HeartConfig::Instance()->SetOutputDirectory("./OUT");
HeartConfig::Instance()->SetOutputFilenamePrefix("results");
HeartConfig::Instance()->SetOutputUsingOriginalNodeOrdering(true);
HeartConfig::Instance()->SetVisualizeWithMeshalyzer(false);
HeartConfig::Instance()->SetVisualizeWithParallelVtk(true);
// PARAMETERS
double Input_Ko
double Input fkatp
                        = 8.5;
                        = 0.05;
double SF Na Ca
                        = 0.75;
double period
                        = 600.0:
MyCellFactory cell factory(Input Ko, Input fkatp, SF Na Ca, period);
// PROBLEM
BidomainProblem<2> bidomain problem( &cell factory );
bidomain problem.SetMesh(&mesh);
bidomain problem.SetWriteInfo();
bidomain problem.Initialise();
bidomain problem.Solve();
```

}



- ✓ Shorter APD values in the ischemic region
- ✓ Activation time map starting from the septum
- ✓ Maximum upstroke velocities slower in the ischemic region



Chaste example 3

• 2D geometry with ischemia and ectopy



Adding the effect of an ectopic beat in the border zone region

```
#include <cxxtest/TestSuite.h>
#include "PetscSetupAndFinalize.hpp"
#include "BidomainProblem.hpp"
#include "RegularStimulus.hpp"
#include "SimpleStimulus.hpp"
#include "TetrahedralMesh.hpp"
#include "ORd2011epi fkatpCvodeOpt.hpp"
class MyCellFactory : public AbstractCardiacCellFactory<2> // <3> here
private:
    boost::shared ptr<RegularStimulus> mpStimulus;
    boost::shared ptr<SimpleStimulus> mpStimulus2;
    double k;
    double fkatp;
    double sf na ca;
public:
    MyCellFactory(float Input Ko, float Input fkatp, float SF Na Ca, float period, float ectopic time)
        : AbstractCardiacCellFactory<2>(),
          mpStimulus(new RegularStimulus(-120000.0, 1, period, 0)),
          mpStimulus2(new SimpleStimulus(-120000.0, 2.0, ectopic time))
    {
        k = Input Ko;
        fkatp = Input fkatp;
        sf na ca = SF Na Ca;
    AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode)
    {
        CellORd2011epi fkatpFromCellMLCvodeOpt* p cell;
        double x = pNode->rGetLocation()[0];
        double y = pNode->rGetLocation()[1];
        if ((x > -0.32) \& (x < -0.27) \& (y > -0.1) \& (y < 0.1))
        {
            p cell = new Cell0Rd2011epi fkatpFromCellMLCvode0pt(mpSolver, mpStimulus);
        }
```

```
public:
    MyCellFactory(float Input Ko, float Input fkatp, float SF Na Ca, float period, float ectopic time)
        : AbstractCardiacCellFactory<2>(),
         mpStimulus(new RegularStimulus(-120000.0, 1, period, 0)),
         mpStimulus2(new SimpleStimulus(-120000.0, 2.0, ectopic time))
    {
        k = Input Ko;
        fkatp = Input fkatp;
        sf na ca = SF Na Ca;
    AbstractCvodeCell* CreateCardiacCellForTissueNode(Node<2>* pNode)
       CellORd2011epi fkatpFromCellMLCvodeOpt* p cell;
        double x = pNode->rGetLocation()[0];
        double y = pNode->rGetLocation()[1];
       if ((x > -0.32) \& (x < -0.27) \& (y > -0.1) \& (y < 0.1))
            p cell = new Cell0Rd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpStimulus);
       else if ((x>-0.4) && (x<0.05) && (y>0.25) && (y<0.44))
            p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);
            p cell->SetParameter("extracellular potassium concentration",k);
            double gna = p cell->GetParameter("membrane fast sodium current conductance");
            double gca = p cell->GetParameter("membrane L type calcium current conductance");
            p cell->SetParameter("membrane fast sodium current conductance".gna*sf na ca);
            p cell->SetParameter("membrane L type calcium current conductance".gca*sf na ca);
            p cell->SetParameter("membrane atp dependent potassium current conductance", fkatp);
        else
            p cell = new CellORd2011epi fkatpFromCellMLCvodeOpt(mpSolver, mpZeroStimulus);
        //Definition Ectopic region
        ChastePoint<2> stim centre (-0.43,0.39);
        ChastePoint<2> stim radius (0.05,0.05);
        ChasteEllipsoid<2> stim region (stim centre, stim radius);
        // Modification of cell properties if it is in the ectopic region
       bool cell is in stim region = stim region.DoesContain(this->GetMesh()->GetNode(pNode->GetIndex())->rGetLocation());
       if(cell is in stim region)
            p cell->SetStimulusFunction(mpStimulus2);
        return p cell;
```



3D simulations in Chaste



- Inclusion of border zones: endocardial and around the ischemic area
- Transmural heterogeneities

Dutta and Minchole et al, PBMB 2016

3D simulations in Chaste

Simulated effect of IKr block in acute ischaemia



Personalization of anatomical models



Zacur, Minchole et al, BIVPCS 2017

Subject DTI003



Computer Simulations to explain Cardiac phenotypes



Linking structure and function HPC cardiac simulation





VI

Example of electrophysiology with Alya

Alya

- Alya functionalities:
 - ✓ Monodomain and bidomain models
 - A few cellular action potential models are implemented (FHN, ORd, TT)
 - Bidirectional electro-mechanical coupling is already implemented and fully functional
 - Excitation-contraction coupling models and models of stretch-activated ion channels are implemented
 - $\checkmark\,$ High efficiency and very high scalability
 - ✓ Other physics are also available (combustion, incompressible and compressible CFD...)

Alya example 1

- Biventricular model (5.2M nodes and 32M elements)
- Run on MareNostrum IV (BSC) on 2,000 cores in 15 min
- This example comprises of 4 input files in ASCII format: .dat, .dom.dat, .ker.dat and .exm.dat
- Electrical propagation only



Electro-mechanical modelling

Equations of motion $\nabla \cdot \mathbf{P} + \rho_0 \mathbf{b} = \rho_0 \ddot{\mathbf{u}}$

Hyperelastic model $\psi(\mathbf{C}) = \frac{a}{2b} \exp[b(I_1 - 3)] + \sum_{i=f,s} \{\exp[b_i(I_{4i} - 1)^2] - 1\} + \frac{a_{fs}}{2b_{fs}} [\exp(b_{fs}I_{8fs}^2) - 1]$ Mechano-electric coupling $\nabla \cdot (J\mathbf{F}^{-1}\mathbf{D}_0\mathbf{F}^{-T}\nabla V) =$ $\chi(C\frac{\mathrm{d}V}{\mathrm{d}t} + I_{ion} - I_{app})$

Electro-mechanic coupling $\frac{d\mathbf{s}}{dt} = m_s(\mathbf{c}, \mathbf{s}, \lambda, \dot{\lambda})$ $T_{act} = h(\lambda) \frac{T_{ref}}{r_s} [(\zeta_s + 1)S + \zeta_w W]$

Active contraction model $\frac{ds}{dt} = m_s(\mathbf{c}, \mathbf{s}, \lambda, \dot{\lambda})$

Active tension generation $T_{act} = h(\lambda) \frac{T_{ref}}{r_s} [(\zeta_s + 1)S + \zeta_w W]$ Stretch-activated ion channels $I_{inw}(V, \mathbf{F}) = f(\sqrt{\lambda} - 1)(V - E)$

Alya example 2

- ✓ Small blob of tissue
- ✓ Run locally with 4 cores
- Electromechanical simulation, Hunter(left) vs Land(right)



Alya .dat and .dom.dat

| \$ RUN_DATA ALYA: heart_test END_RUN_DATA | | | | | | |
|--|--------|---------|-------|------|------|------|
| PROBLEM DATA | | | | | | |
| MAXIMUM NUMBER GLO | OBAL: | 1 | | | | |
| NUMBER_OF_STEPS: | | 1000000 |) | | | |
| TIME_INTERVAL: | | 0.000, | 0.400 | | | |
| \$ | | | | | | |
| EXMEDI: | ON | | | | | |
| END_EXMEDI | | | | | | |
| \$SOLIDZ_PROBLEM: | ON | | | | | |
| \$END_SOLIDZ | | | | | | |
| PARALL_SERVICE: | ON | | | | | |
| OUTPUT_FILE: | NO | | | | | |
| POSTPROCESS: | MASTER | | | | | |
| PARTITION: | FACES | | | | | |
| END_PARALL | | | | | | |
| \$ | | | | | | |
| END_PROBLEM_DATA | | | | | | |
| \$ | | | | | | |

\$-----DIMENSIONS NODAL POINTS = 5196552 ELEMENTS = 31702122 SPACE DIMENSIONS = 3 NODES = 4 BOUNDARIES = 920090 END_DIMENSIONS \$-----STRATEGY INTEGRATION RULE = OPEN DOMAIN INTEGRATION POINTS = 1 END_STRATEGY \$-----GEOMETRY, GID, WALL DISTANCE = 0.0, ROUGHNESS = 0.0 FIELDS, NUMBER = 3 FIELD = 1, DIMENSION = 3, NODES INCLUDE heart test.fiber END FIELD FIELD = 2, DIMENSION = 1, NODES INCLUDE heart test.celltype END FIELD INCLUDE heart test.stimuli END FIELDS INCLUDE heart test.geo INCLUDE heart test.surfaces END GEOMETRY \$-----BOUNDARIES, EXTRAPOLATE \$ INCLUDE heart test.boundaries END BOUNDARIES

Alya .ker.dat and .exm.dat

\$----PHYSICAL PROBLEM \$COUPLING \$ SOLIDZ EXMEDI \$END COUPLING END PHYSICAL PROBLEM \$-----NUMERICAL TREATMENT MESH DIVISION = 1\$ END MESH END_NUMERICAL_TREATMENT \$-----OUTPUT & POST PROCESS ON LAST MESH STEPS = 50END OUTPUT & POST PROCESS \$----- PHYSICAL PROBLEM \$-----PROBLEM DEFINITION STARTING POTENTIAL, FIELD CURDENSITY: FIELD = 3 END STARTING POTENTIAL \$GEO COUPLING: LAGRANGIAN END PROBLEM DEFINITION \$-----PROPERTIES MATERIAL = 1 CONTINUUM MODEL IN DIFFUSION = 0.00107143 0.00024107 0.00024107 [cm2/ms] INITIAL MEMBRANE POTENTIAL= -87.0 [mV] END CONTINNUM MODEL CELL MODEL: OHARA CM CONST = 1.0ENDCELL MODEL INI CELL, HETEROGENEOUS INCLUDE heart test.framework END INI CELL END PROPERTIES \$-----..... FIBER MODEL: FIELD = 1 CELL TYPE: FIELD = 2 \$----END PHYSICAL PROBLEM \$-----NUMERICAL TREATMENT \$CONSTANT MATRIX SAFETY FACTOR = 25.0 TIMETREATMENT: IMPLICIT, CRANK ALGEBRAIC SOLVER SOLVER: CG CONVERGENCE : ITERA=1000, TOLER=1.0e-6, ADAPTIVE, RATIO=0.01 OUTPUT: CONVERGENCE PRECONDITIONER: DIAGONAL END ALGEBRAIC SOLVER END NUMERICAL TREATMENT \$-----OUTPUT & POST PROCESS POSTPROCESS INTRA END OUTPUT & POST-PROCESS \$-----

\$-----

Final comments

- Complex nature of biology:
 - Models as tools to augment experimental/clinical findings.
- ▶ <u>HPC</u> is needed for whole biventricular and torso simulations:
 - Models can several tens of millions of elements!
- Highlight of the potential of mathematical modelling for <u>in silico</u> <u>clinical and drug trials</u>:
 - Substitution of current protocols for human *in silico* protocols
- Highlight of the potential of mathematical modelling for explaining the <u>underlying mechanisms of abnormalities in the ECG</u>:
 - Potential improvement of the associated inverse problem

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wellcometrust









European Commission