Imposing Ratios of Outlet Flow Rates on Large Arterial Networks with Two-Element Windkessel Model: Parametric Analysis

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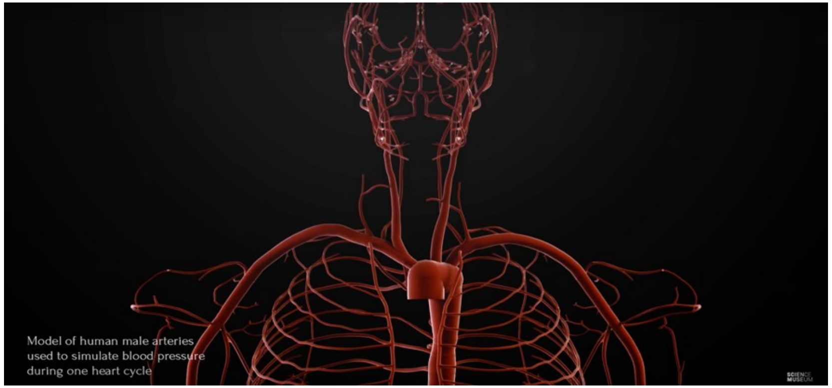
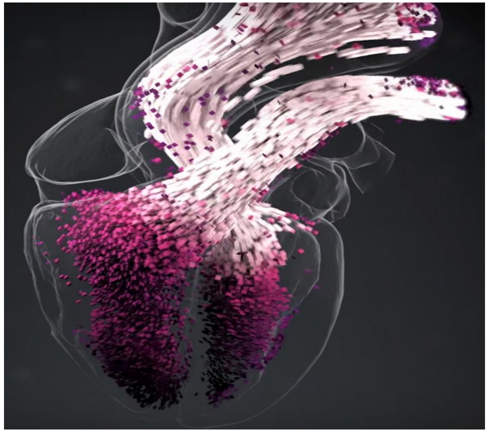
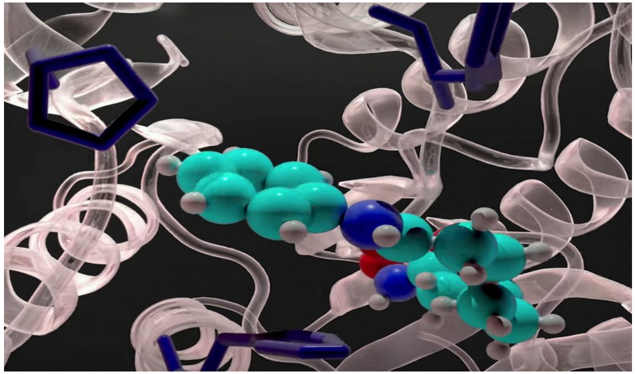
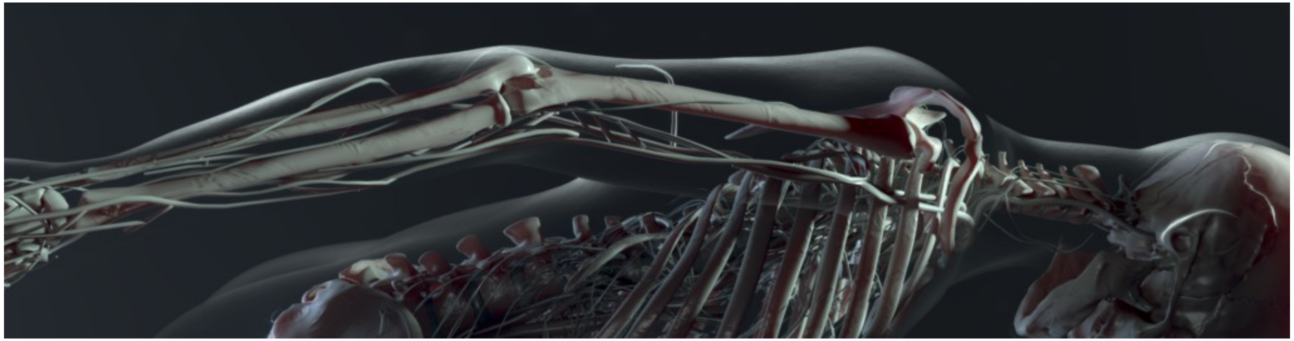
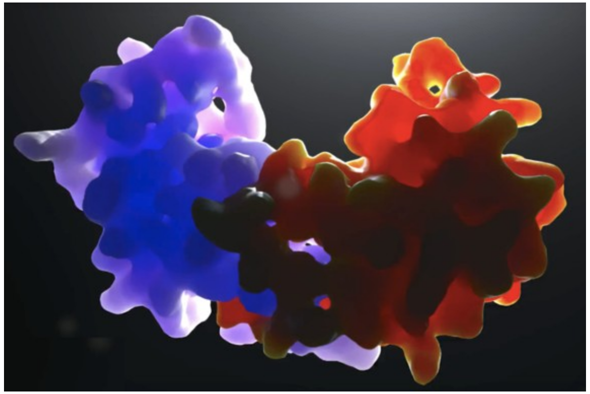
Virtual Physiological Human











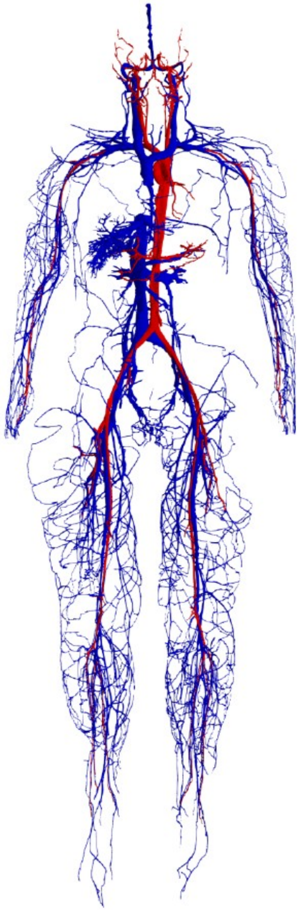
- Will improve our understanding of human physiology and diseases
- Will assist clinicians in the design of personalised medical treatments

Source: CompBioMed

Human Circulatory System

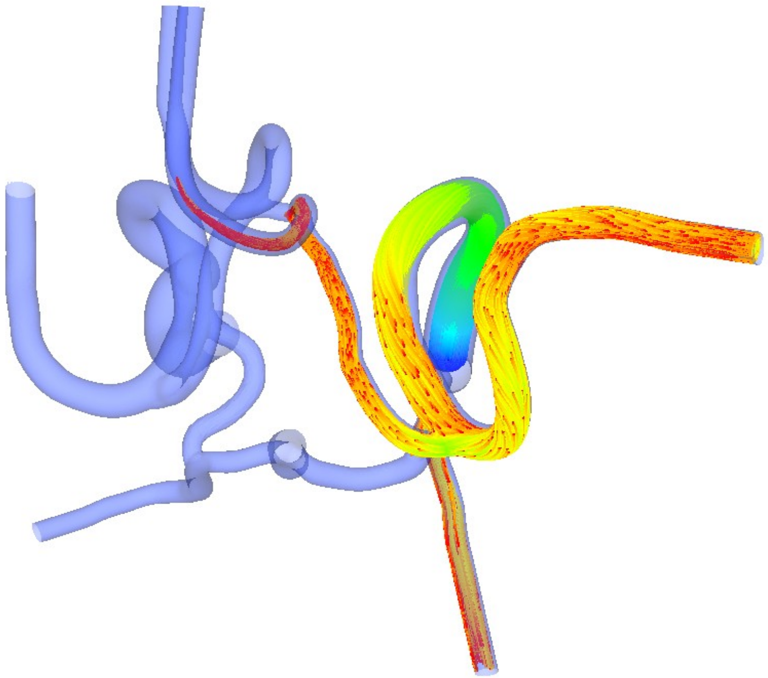
Characteristics:

- Highly complex geometries
- Highly unsteady processes
- Different dynamic scales
- Fluid-structure interactions



Source: CompBioMed

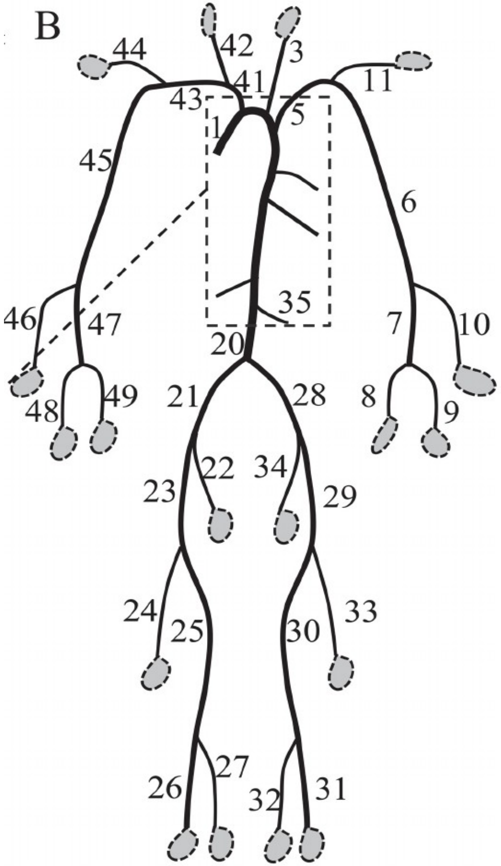
HemeLB



- 3D fluid flow solver based on the lattice Boltzmann (LB) method
- Suitable for complex geometries
- Highly scalable on CPUs and GPUs
- Available features include
  a) self-coupling,
  b) elastic wall model,
  c) rheology models, etc.
- Open-source

Source: www.hemelb.org

Imposing Boundary Conditions

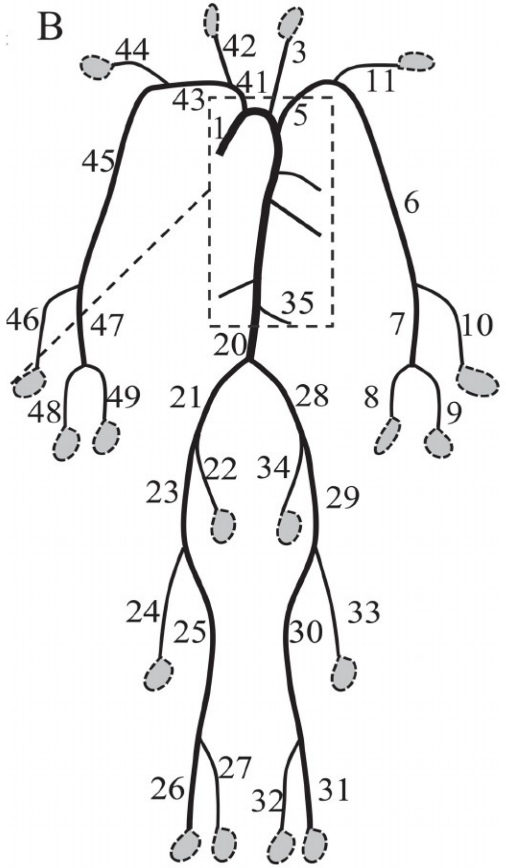


Aims:
- Accurately represent the truncated systems
- Efficient and scalable

Du T, Hu D, Cai D (2015) Outflow Boundary Conditions for Blood Flow in Arterial Trees. PloS ONE 10(5): e0128597. doi:10.1371/journal. pone.0128597

Int J Adv Eng Sci Appl Math (January–March 2016) 8(1):46–60

Imposing Boundary Conditions



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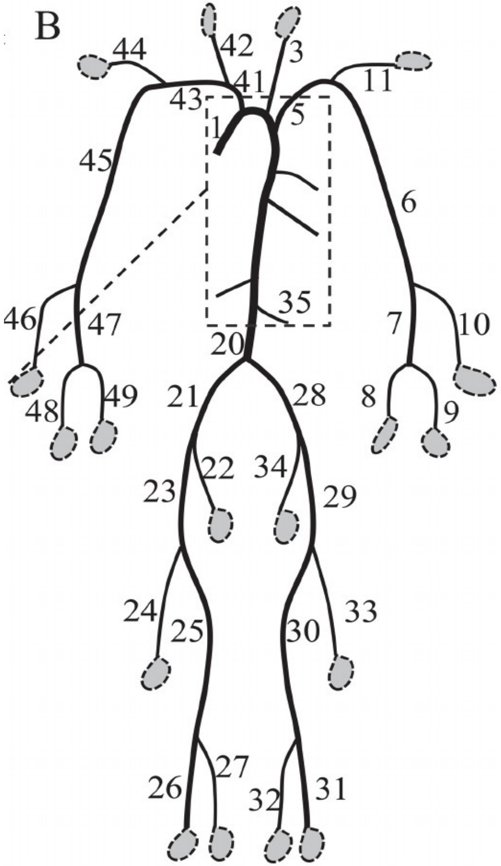
Common strategies:

- Pressure
- Traction
- Flow rate + assumptions on flow profile
- Coupling with reduced-order models + parameters tuning

Imposing Boundary Conditions



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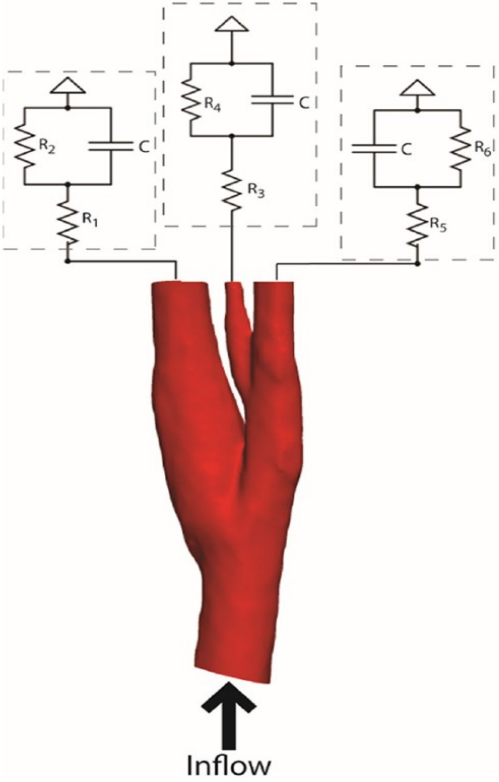

Imposing Flow Rate Ratios at Outlets

Grinberg's strategy:

- Prescribes ratios of flow rates (Q) at outlets
- Efficient for large systems

$$\frac{Q_{\mathrm{ref}}}{Q_j} \approx \frac{R_j}{R_{\mathrm{ref}}} \equiv \eta_j$$

where j = 1, …, N if there are N outlets, and *ref* refers to one of the outlets.

Grinberg, L. & Karniadakis, G. E. Outflow boundary conditions for arterial networks with multiple outlets. Annals Biomed. Eng. 36, 1496–1514, DOI: 10.1007/s10439-008-9527-7 (2008).

Imposing Flow Rate Ratios at Outlets

Grinberg’s strategy:

- Prescribes ratios of flow rates (Q) at outlets
- Efficient for large systems
- Accurate if resistances (R) are sufficiently high

$$\frac{Q_{\mathrm{ref}}}{Q_j} \approx \frac{R_j}{R_{\mathrm{ref}}} \equiv \eta_j$$

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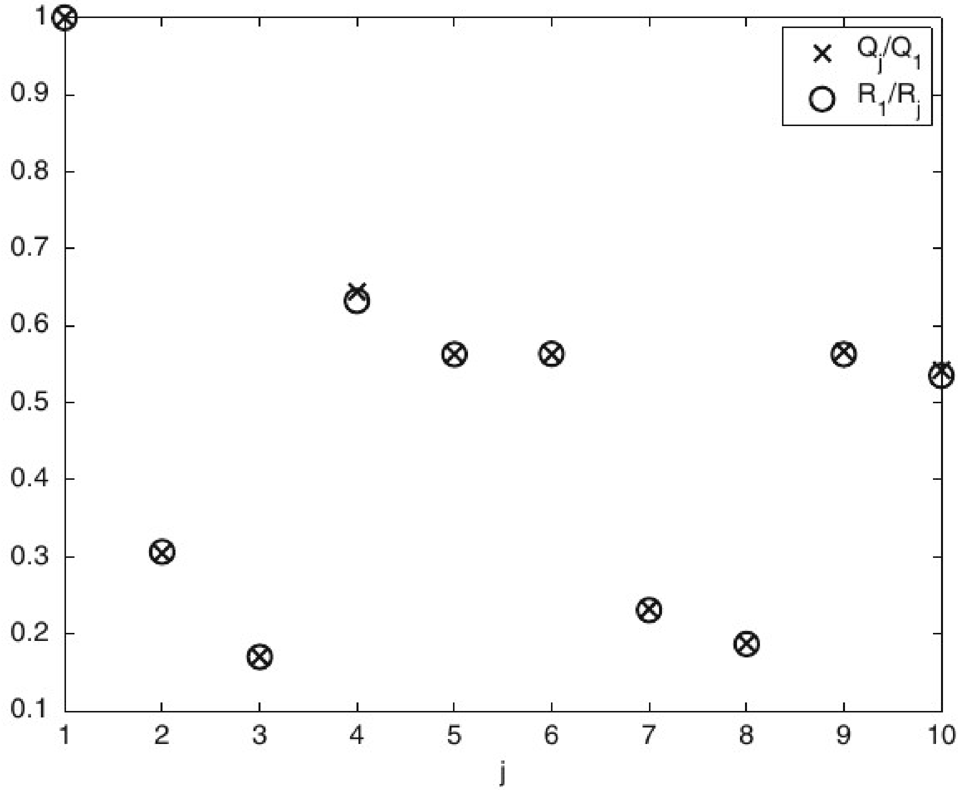

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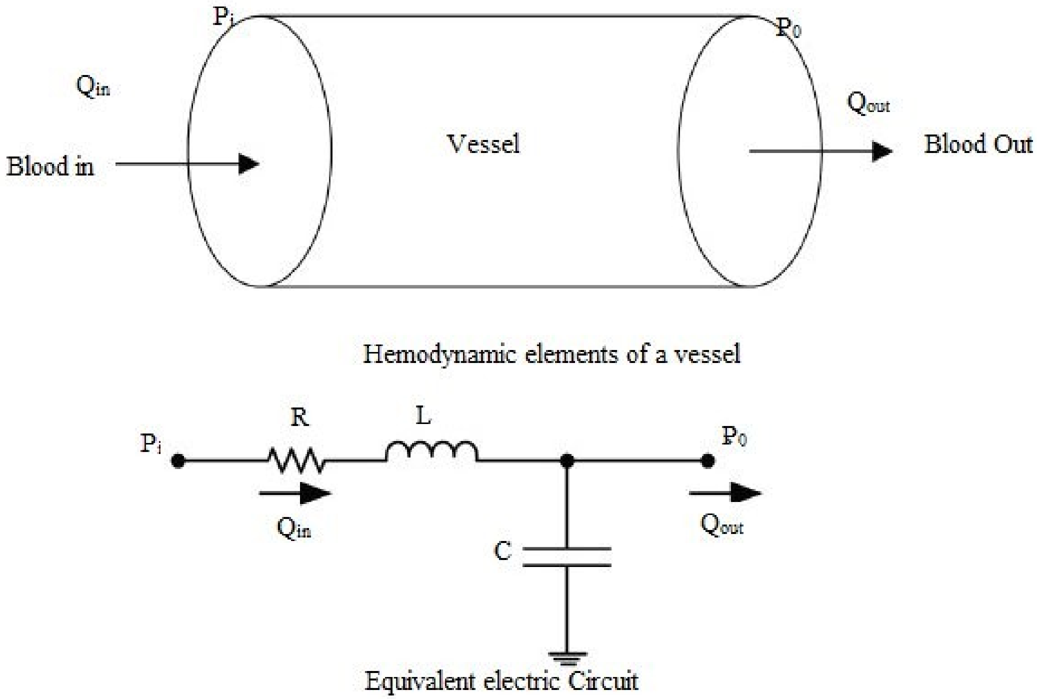
Contributions of the Current Effort

- Clarify the general role of the parameters (R and C)
- Study the impacts of the parameters on the stability, accuracy, and convergence of the simulations

Sharp Chim Yui Lo, J. W. S. McCullough, P. V. Coveney et al. Imposing Ratios of Outlet Flow Rates on Large Arterial Networks with Two-Element Windkessel Model: Parametric Analysis, 10 May 2022, PREPRINT (Version 1).

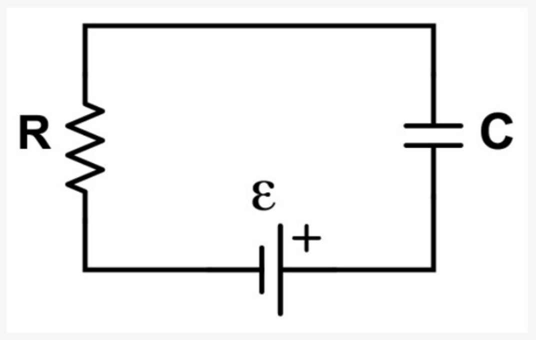
Electric Circuit Analogy

Blood Vessel	Electric Circuit
pressure	voltage
flow rate	electric current
flow resistance	resistance
wall elasticity	capacitance
flow inertia	inductance



Naik, Ketan B., and P. H. Bhathawala. "Mathematical modelling and simulation of human systemic arterialsystem." Int J Eng Innovative Technol 4, no. 1 (2014)

Two-Element Windkessel Model



- Approximates vessels as an RC circuit
- Converts flow velocity to pressure at boundaries
- Filters high-frequency noise

$$P + RC\frac{dP}{dt} = RQ$$

$$Q = \int \vec{U} \cdot \hat{n}\, dA$$

P: pressure

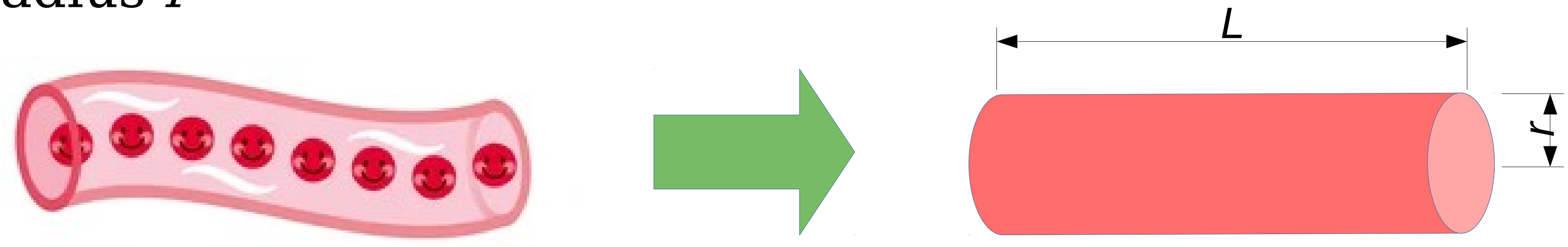
U: flow velocity

Q: flow rate

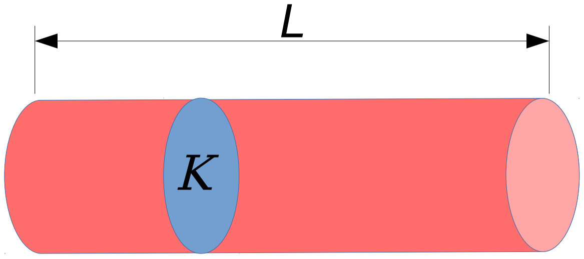
Source: https://www.quora.com/What-is-the-impedance-of-an-RC-circuit-connected-across-DC-source

Choice of Resistance

1. Conceive an arbitrary vessel as a cylinder of length L and radius r



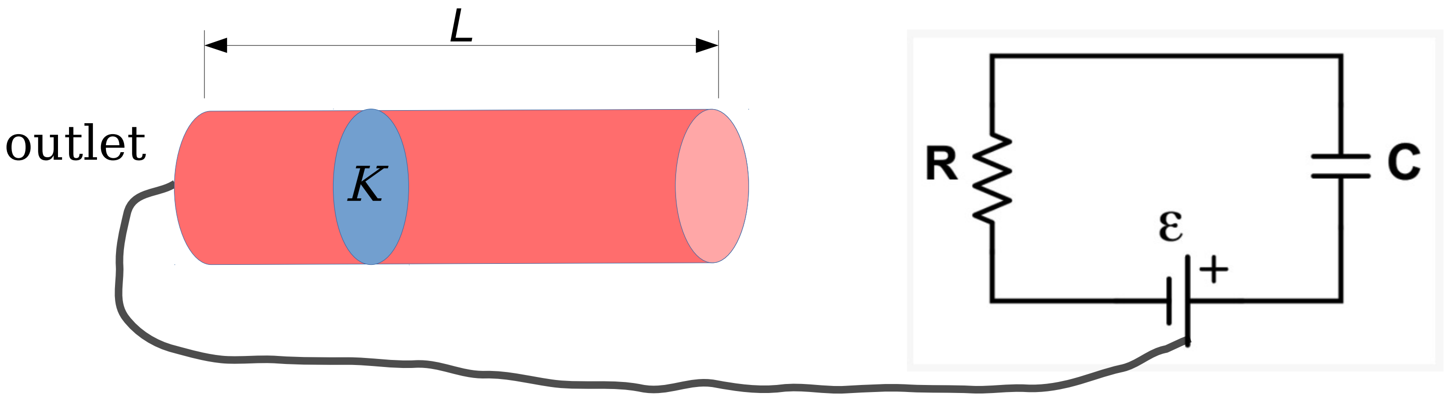
2. Then, the flow resistance of the vessel is LK, where $K = \dfrac{8\mu}{\pi r^4}$



Source: https://www.shutterstock.com/image-illustration/illustration-blood-vessel-301073339

Choice of Resistance

3. The total resistance of an outlet vessel is given by $LK + R$



4. Then, a necessary condition for Grinberg's method must be

$$R \gg LK$$

Recall $\frac{Q_{\mathrm{ref}}}{Q_j} \approx \frac{R_j}{R_{\mathrm{ref}}} \equiv \eta_j$

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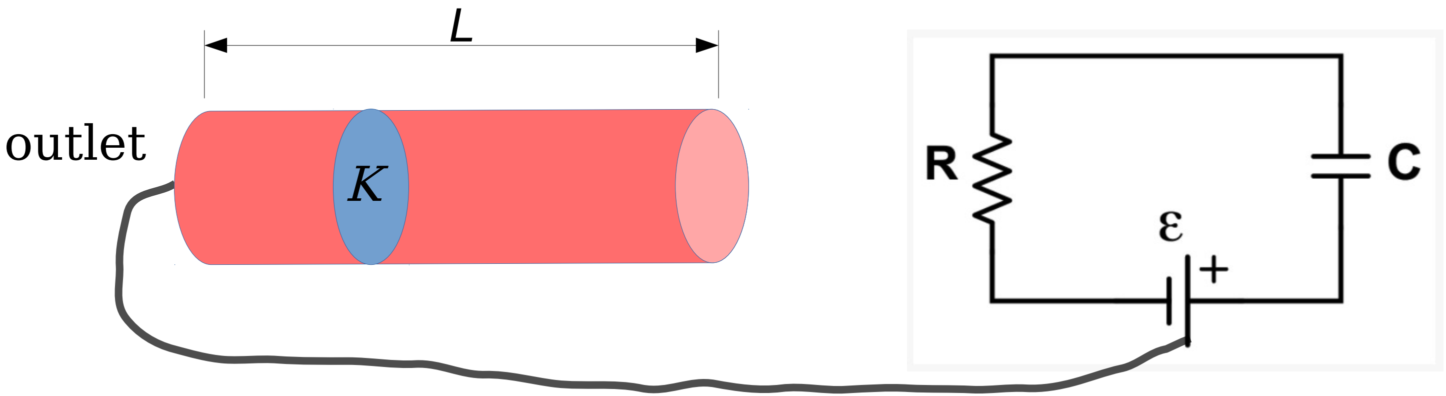

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Recall $\frac{Q_{\mathrm{ref}}}{Q_j} \approx \frac{R_j}{R_{\mathrm{ref}}} \equiv \eta_j$

5. This condition is parametrised as

$$\boxed{R_{\mathrm{ref}} = \gamma_R \hat{L} \hat{K}}$$

Choice of Capacitance

Upper limit for the capacitance:

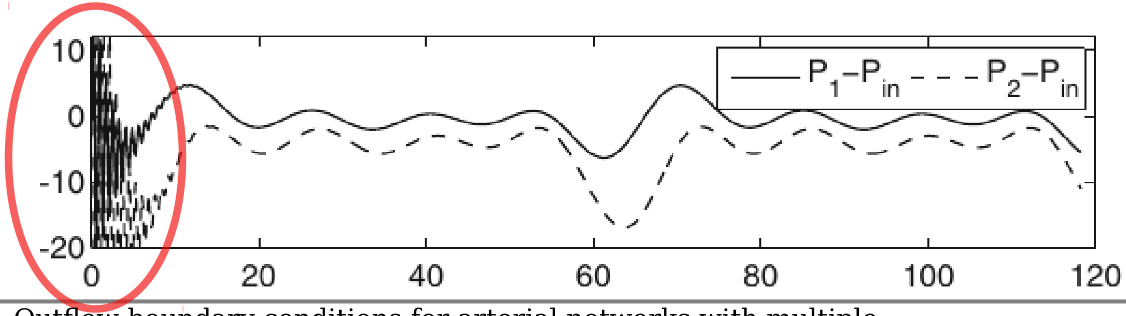
1. The homogeneous (transient) solution is

$$P(t) = P(0)\exp(-t/RC)$$

2. The characteristic decay time is RC

Recall

$$P + RC\frac{dP}{dt} = RQ$$



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Choice of Capacitance

Upper limit for the capacitance:

1. The homogeneous (transient) solution is

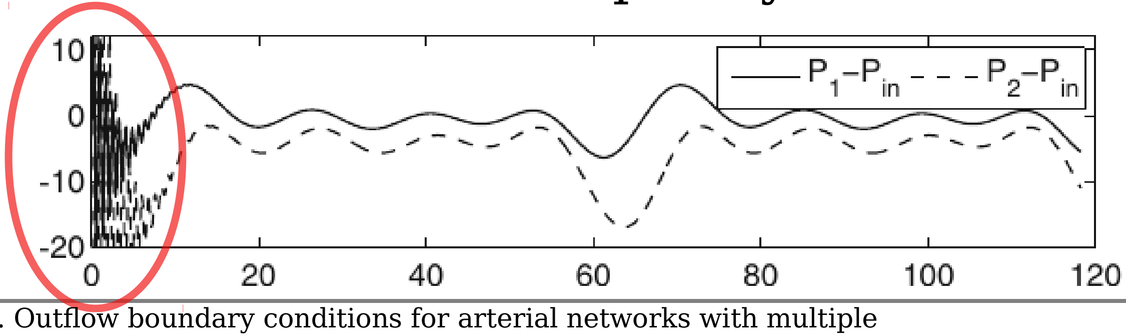
$$P(t) = P(0)\exp(-t/RC)$$

2. The characteristic decay time is RC
3. Transient solutions should fade out shortly
4. Therefore, we should **avoid** $RC \gg 1/\omega_0$

   where ω_0 is the fundamental frequency

Recall

$$P + RC\frac{dP}{dt} = RQ$$



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Choice of Capacitance

Lower limit for the capacitance:

1. In Fourier space,

$$\hat{P}(\omega) = Z(\omega)\hat{Q}(\omega),$$

$$Z(\omega) = \frac{R}{1+i\omega RC} = \frac{R}{\sqrt{1+(\omega RC)^2}}\exp[i\underline{\tan^{-1}(-\omega RC)}]$$

Recall

$$\boxed{P + RC\frac{dP}{dt} = RQ}$$

2. The pressure and the velocity have a phase difference of

$$\tan^{-1}(-\omega RC)$$

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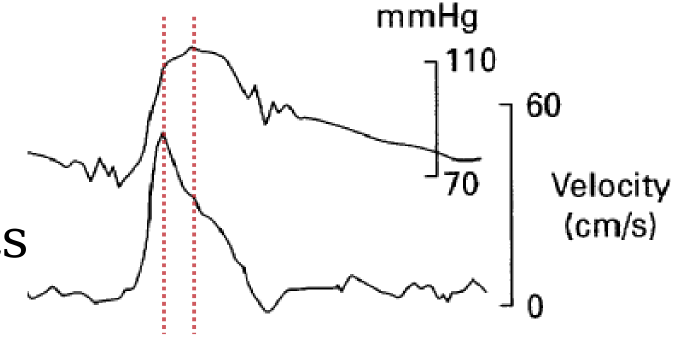
3. Such a time lag is also observed in experiments



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suggest a parametrisation

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The upper limit and the lower limit

$$RC \gg 1/\omega_0 \qquad\qquad RC \ll 1/\omega_0$$

suggest a parametrisation:

$$\boxed{C_j = \gamma_{RC}/(\omega_0 R_j)}$$

Desired Flow Rate Ratios

- Based on the principle of minimum work, Murray showed that $Q \sim r^3$ in any cross-section of a vessel
- With mass conservation, the relation becomes

$$Q_j = \frac{r_j^3}{\sum_j r_j^3}$$

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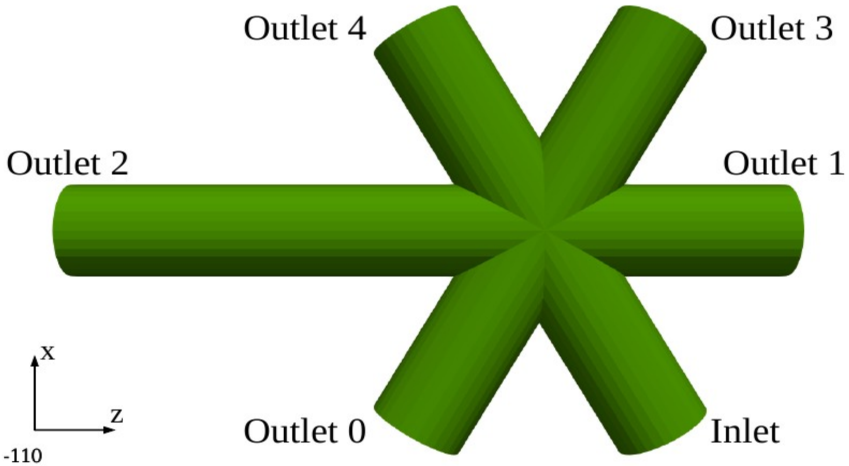
- With mass conservation, the relation becomes

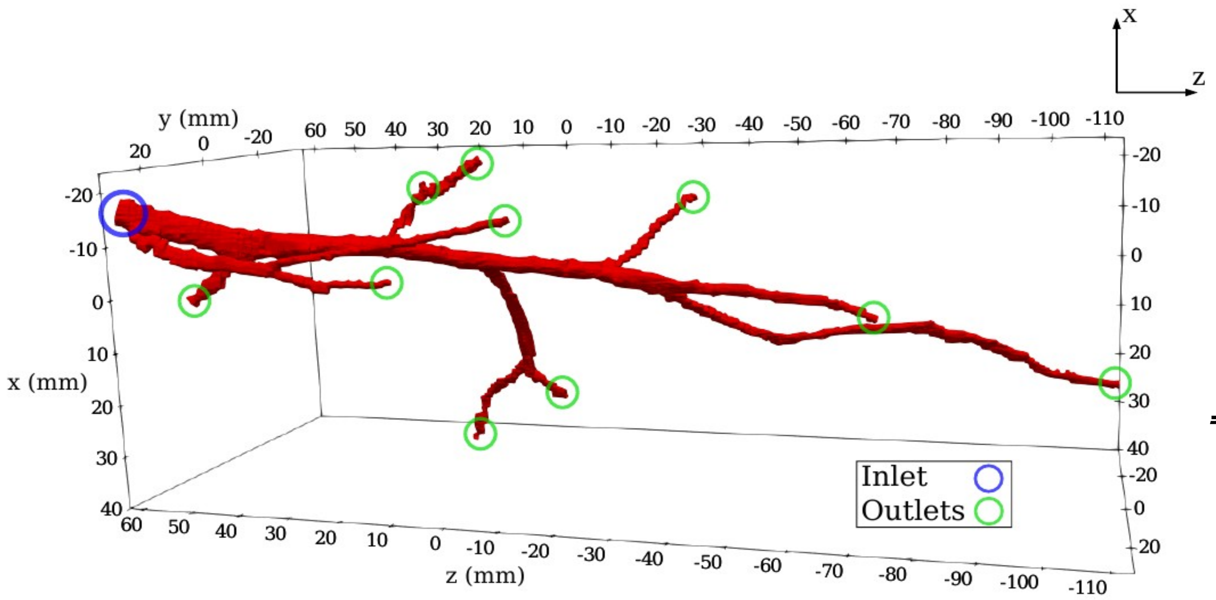
$$Q_j = \frac{r_j^3}{\sum_j r_j^3}$$

- Depends only on the geometry used
- Useful when physiological data is unavailable
- For a more accurate description, other powers can be used

Simulation Domains



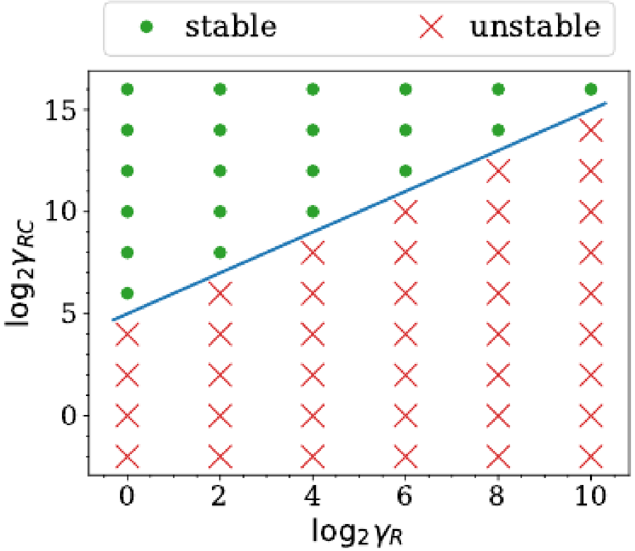
five-outlets



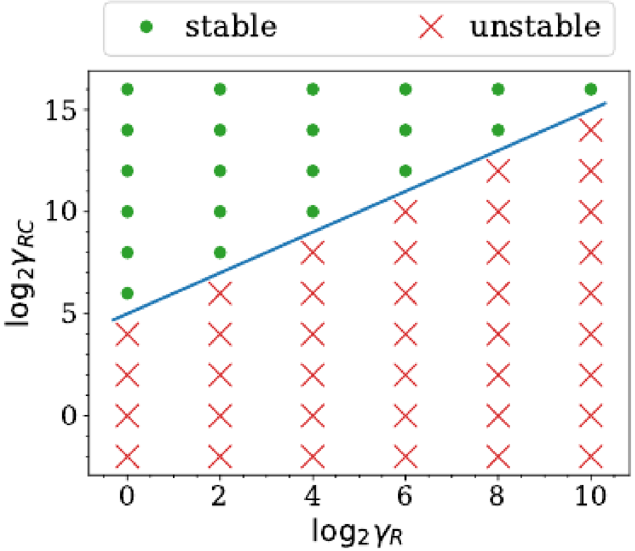
profunda femoris

Stability Maps

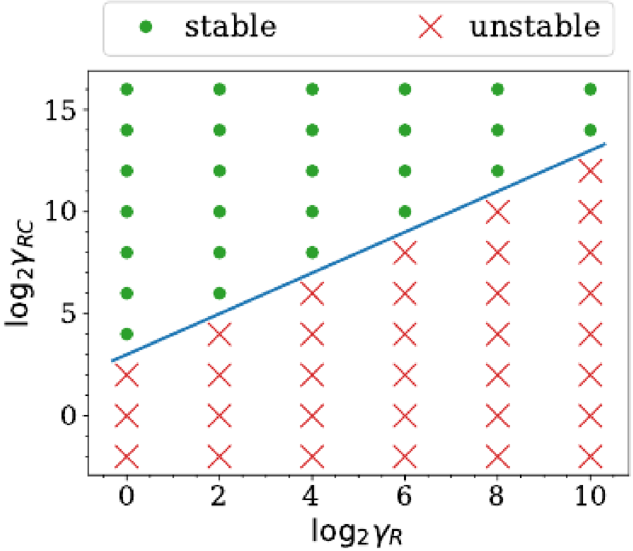
- Model parameters cannot be arbitrary
- Stability enhances with the value of C



Coarse grid



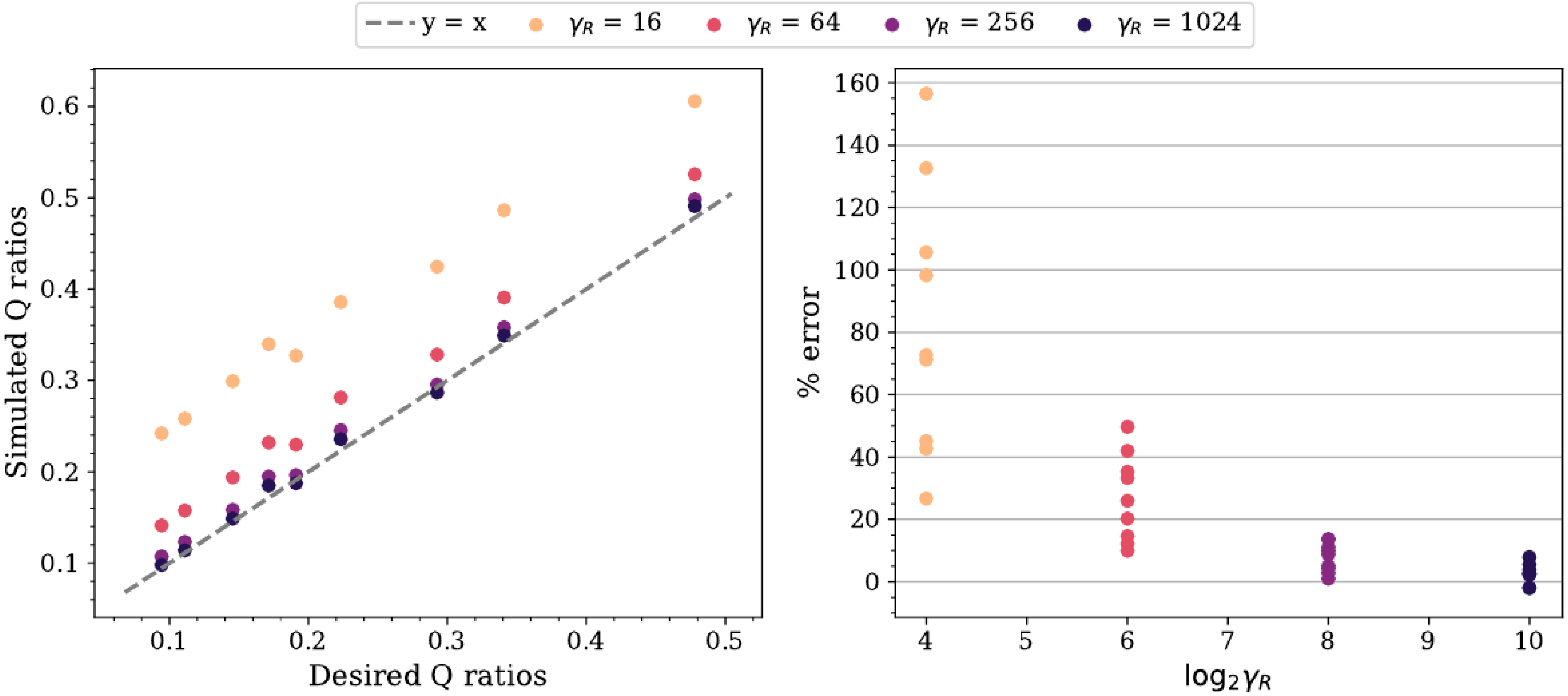
Medium grid



Fine grid

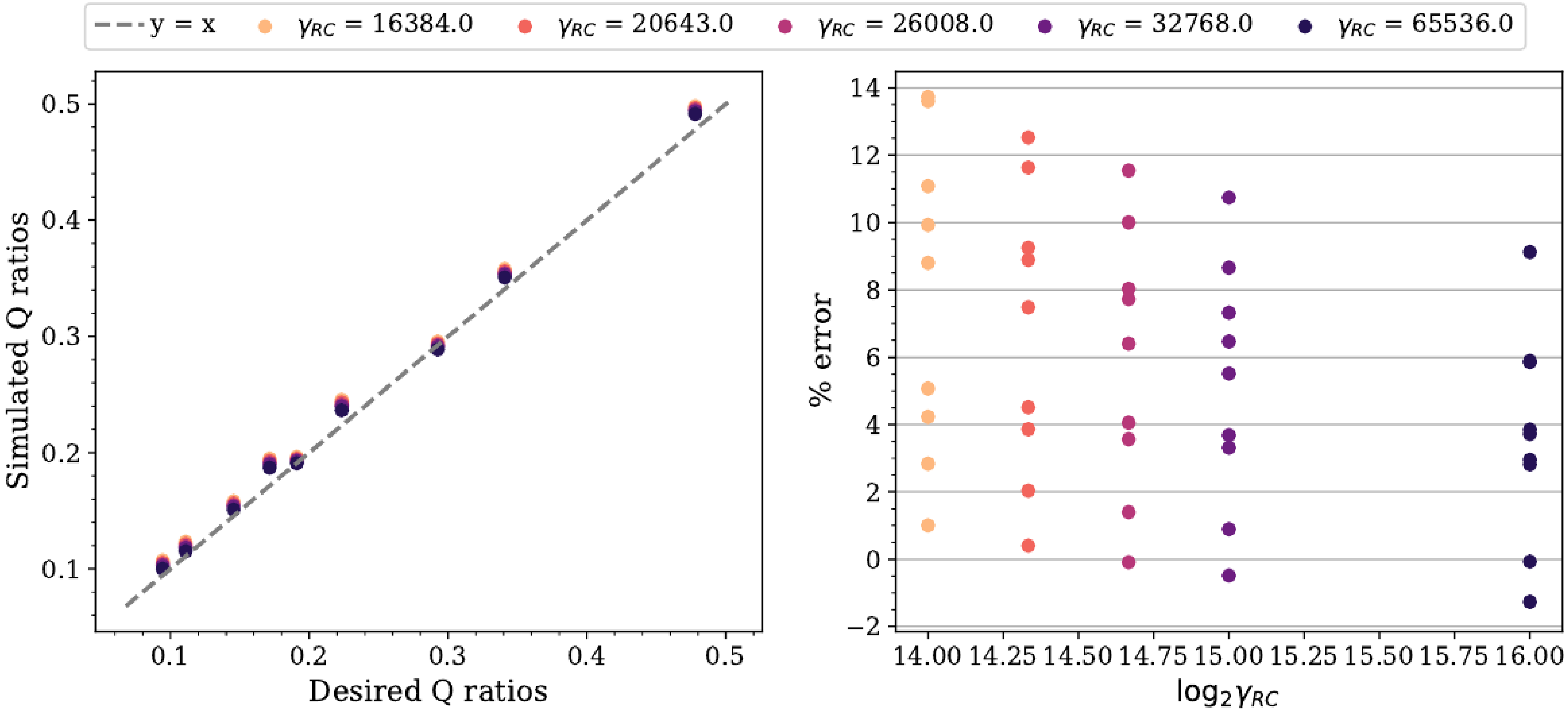
Error of Flow Rate Ratios

- Q ratios approach the desired values as R increases
- But there are intrinsic errors



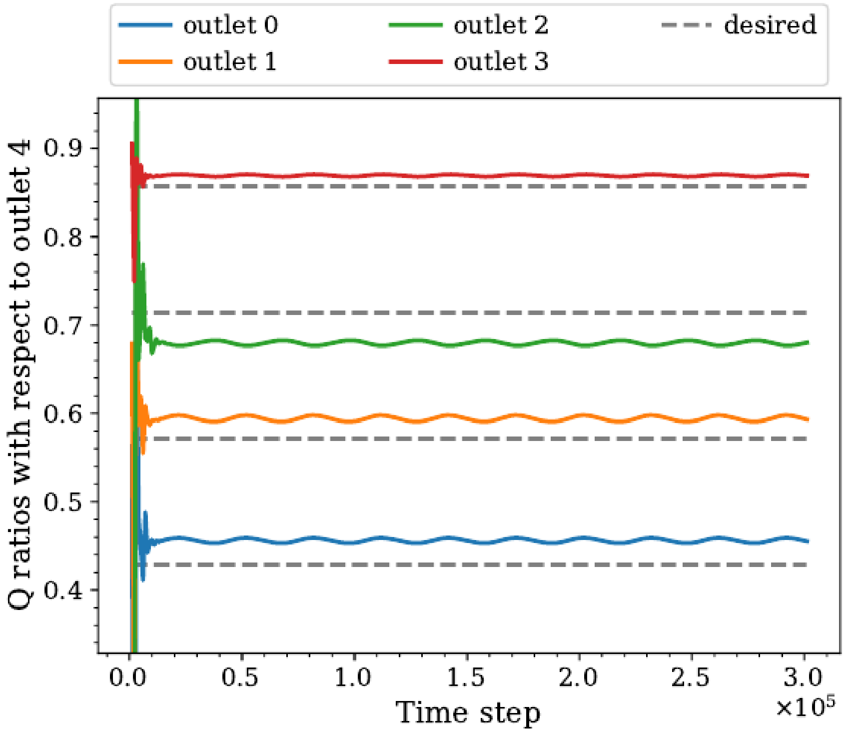
Error of Flow Rate Ratios

- The Q ratios depend weakly on C

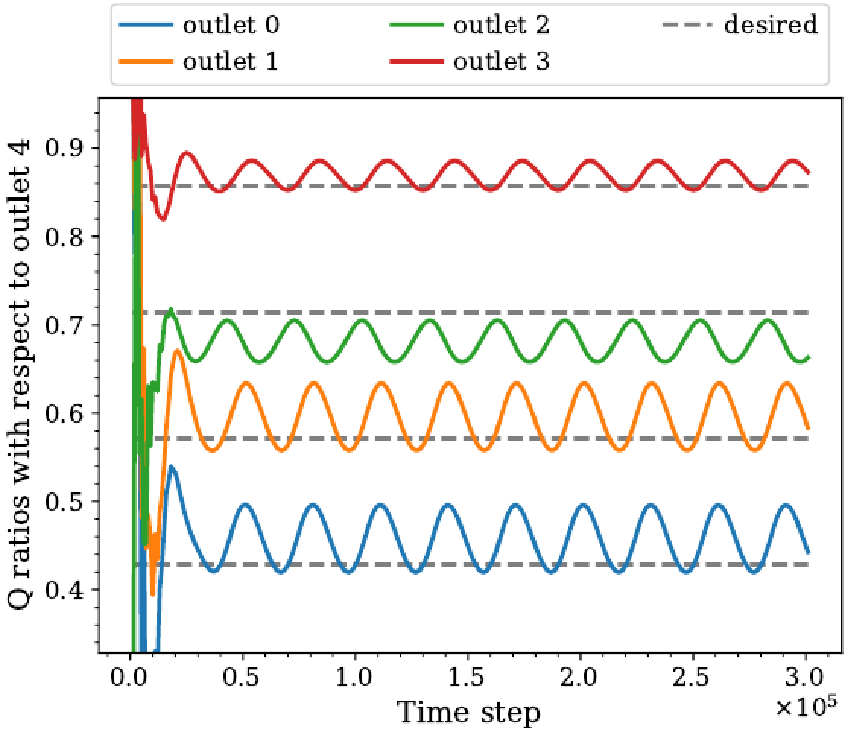


Convergence and Fluctuations

- Slower convergence and stronger fluctuation as C increases

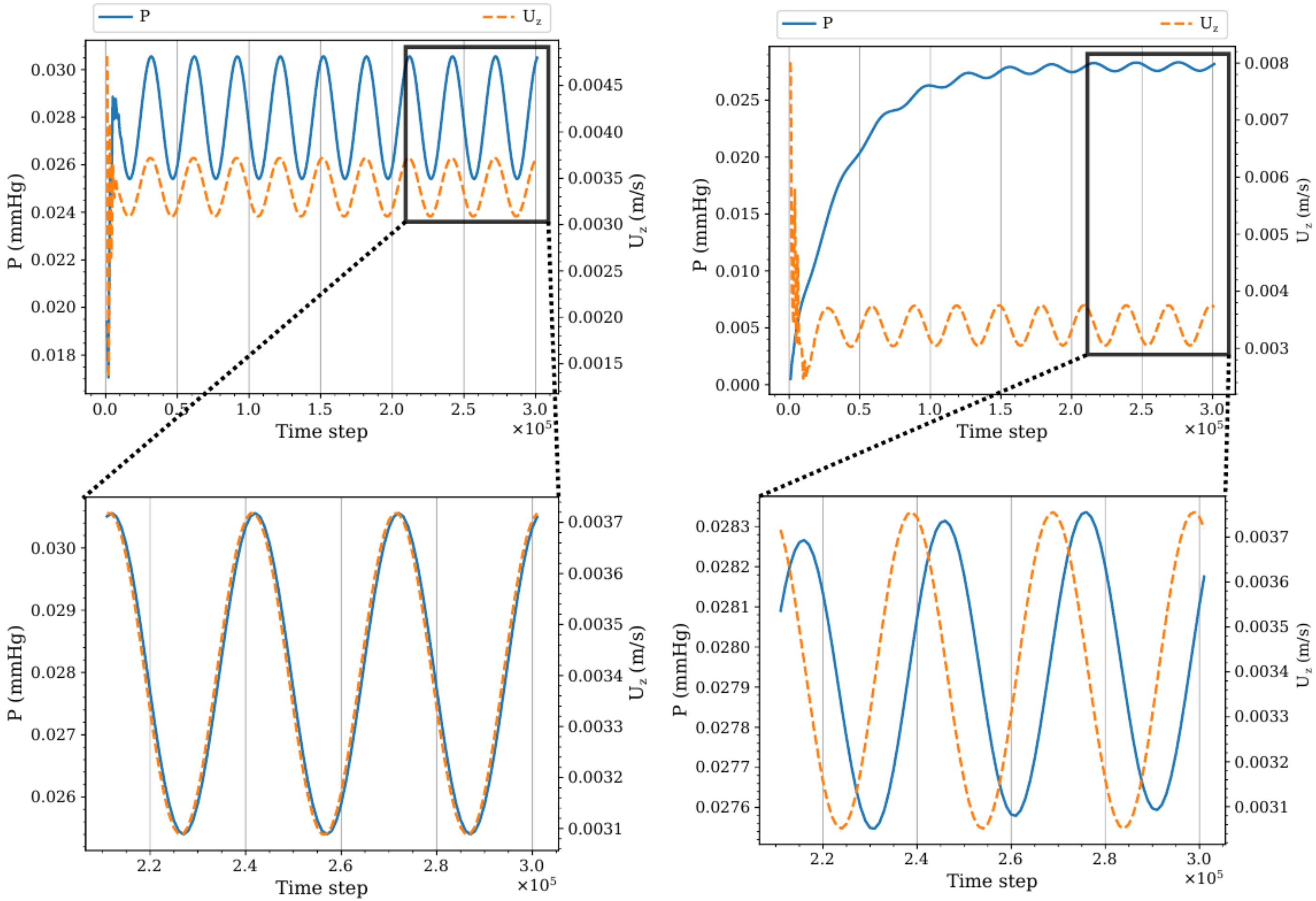


Smaller C



Larger C

Time lag



- Larger time lag as C increases

<u>Left: Smaller C</u>

<u>Right: Larger C</u>

Conclusions

- The value of R should be larger than a threshold
- An estimation of the threshold value is proposed and tested

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Conclusions

- The value of R should be larger than a threshold
- An estimation of the threshold value is proposed and tested
- The smaller the value of C, the faster the convergence rate and the weaker the fluctuation of flow variables
- The simulation is stable if C is larger than a threshold
- Guideline for choosing the values of R and C
- The methods used are directly applicable to larger and more complex vascular domains encountered at full-human scale

Acknowledgements

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